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The Hydrodynamic Model of High Energy Collision

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Abstract : A high energy collision between two protons, or, in general, two heavy ions, results in the production of a multitude of secondaries, most of which are pions.The average number of secondaries produced in a specified type of collision varies in a smooth way with the collision energy,while for a given energy the number (usually called multiplicity) varies randomly from event to event. The energy and momentum distributions of the secondaries are also of considerable interest. Several models have been proposed over the last forty years to explain the observed phenomena.The hydrodynamic model is one of the simplest and most successful models.

Key words: High energy collision, Hydrodynamics, Particle production, Multiplicity fluctuations.

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1. Introduction

This article is dedicated to Professor Haridas Banerjee whose sixtieth birthday was observed recently. I consider it my good fortune that I could benefit from his advice while working towards my Ph.D thesis. Indeed a number of key ideas presented in the thesis were suggested by him. In this article I shall firstly summarize my work with Prof. Banerjee and then, towards the end, I shall mention what extensions and modifications of that work we are making at present.

The hydrodynamic model of high energy collisions was proposed by L.D.Landau in 1953 [1]. The model was originally advanced for pp collisions and was subsequently extended to heavy ion collisions. We shall describe the model in more or less contemporary language. The collision between two protons or two heavy ions is supposed to release the fundamental constituents of matter, namely quarks and gluons(collectively called partons). Various estimates based on QCD (Quantum Chromodynamics) show that the mean free path of the partons is small compared to the physical dimensions of the post-collision system [2]. Hence the post-collision state of the colliding matter can be described in terms of hydrodynamics supplemented by thermodynamics.

The usual assumption is that the fluid is ideal, so that the energy-momentum tensor $T_{\mu\nu}$ is given by

$$T_{\mu\nu} = (\epsilon + P)u_\mu u_\nu - g_{\mu\nu}P, \tag{1}$$

ϵ being the energy density and P the pressure; u_μ are the components of the four velocity of a fluid element. The equations of motion are

$$\frac{\partial T_{\mu\nu}}{\partial x_\iota} = 0 \tag{2}$$

and, for an ideal fluid, entropy is conserved, so that we also have

$$\partial^\mu s_\mu = 0 \tag{3}$$

where $s_\mu = su_\mu$, s being the entropy density. It is further assumed that the equation of state is $P = c_s^2\epsilon$, c_s being the velocity of sound in the fluid. ($c_s^2 = 1/3$ for an ideal fluid.)

The solution of the equations of motion depends on the initial conditions. Two initial conditions have been considered in the literature,leading to somewhat different solutions. They are summarised below.

A.The Landau-Khalatnikov solution:

Here it is assumed that the two colliding particles completely stop each other, so that $u_\mu = 0$ initially [1]. The resulting fluid body is called a fireball. Because of the large pressure gradient in the longitudinal direction (along the collision axis), the fluid expands predominantly in this direction and an approximate treatment addressing only the longitudinal motion is more or less adequate. The approximate solutions for velocity v and temperature T, in terms of the initial temperature T_0 and initial size Δ, are

$$v = tanh\eta \tag{4}$$

where η is the pseudo-rapidity,

$$\eta = \alpha - \frac{1}{4}\frac{\beta+\alpha}{\beta-\alpha} \tag{5}$$

and

$$ln(\frac{T}{T_0}) = -[\frac{1+c_s^2}{2}\beta - \frac{1-c_s^2}{2}\sqrt{\beta^2-\alpha^2}] \tag{6}$$

where

$$\alpha = \frac{1}{2}ln\frac{t+x}{t-x}, \qquad \beta = \frac{1}{2}ln\frac{t^2-x^2}{\Delta^2} \tag{7}$$

t and x being the time and the longitudinal coordinate.

B.The Chiu-Sudarshan-Bjorken solution:

In this model [3] it is assumed that the partons free-stream after collision upto a proper time τ_0. The solutions for v and T are

$$v = \frac{x}{t} \tag{8}$$

and

$$T = T_0(\tau/\tau_0)^{-c_s^2}, \qquad (\tau = \sqrt{t^2-x^2}) \tag{9}$$

2. Statistical thermodynamic consideration

In the hydrodynamic model contact between thermodynamics and observed quantities, specifically the number of charged secondaries produced, is established through the relation $< N_{ch} > \propto S$, where S is the total entropy of the system. In actual computations one deals with the partition function Z of the whole system. This (for each species) is given by

$$lnZ = \mp g\int d\bar{v}\int\frac{d^3p}{(2\pi)^3}ln[1\mp exp(\mu_c-\bar{E})/T] \tag{10}$$

where μ_c is the chemical potential(equal to zero in the present case), $\bar{E}$ and $\bar{v}$ are respectively the energy of a particle and the volume of a fluid element in the local rest frame and g is the statistical weight factor. The($\mp$) signs in the expression correspond to bosons and fermions respectively. The average multiplicity of the species concerned is given by

$$< N >= T[\frac{\partial}{\partial \mu_c}(lnZ)]_{\mu_c=0} \tag{11}$$

Furthermore, the single particle inclusive spectrum is given by

$$\frac{E}{\sigma}\frac{d\sigma}{d^3p} \quad \frac{g}{(2\pi)^3}\int d\Sigma_\mu p^\mu \frac{1}{exp(\bar{E}/T) \pm 1}, \tag{12}$$

where Σ is the hypersurface over which the integration is to be carried out and σ is the collision cross-section.

Actually the model presented above requires some modifications. The fact is that the thermalised system inherits only a fraction $k(\sim 0.5)$ of the c.m. energy, the rest going away with two subsystems which move rapidly along the beam directions. These subsystems give rise to a few secondaries, but principally a pair of particles, whose quark compositions basically match those of the colliding particles. These are known as the leading particles, because they move with the highest speeds.

3. Banerjee's model

It was suggested by Banerjee that if the leading particles are subtracted out from the collection of secondaries produced in a hadron-hadron collision then the energy-momentum distribution of the remaining secondaries may match the energy-momentum distribution of secondaries generated in e^+e^- annihilation at the same available energy. This was subsequently established by Basile et al [4]. Since it was known that the non-leading secondaries could be described to some degree of precision in terms of Landau's hydrodynamic model, Banerjee, Biswas and De [5] applied the hydrodynamic model to e^+e^- annihilation and achieved a reasonable degree of success. To explain certain features of the data (e.g.the growth of $< p_T^2 >$with W,the available energy)they had to make some modifications in the original model; namely, instead of the freeze-out criterion of Landau (according to which secondary hadrons are produced from a fluid element when its temperature falls to $m_\pi(\sim 140 MeV)$, they adopted the instantaneous disintegration model in which the fireball disintegrates into secondaries when its size reaches a critical value (Fig.1).

4. Results

1. The Banerjee-Biswas-De model, having been successful in explaining the e^+e^- annihilation data, was then adopted to describe the secondaries produced in the central region (i.e. the non-leading secondaries) in pp and $p\bar{p}$ collisions [6]. According to the similarity hypothesis the spectra of secondaries for the two reactions should match at the same value of the available energy. It was found necessary to assume that three fireballs are produced in a hadron-hadron collision. One of these, the central fireball, decays like the fireball in e^+e^- annihilation. The other two give rise to the leading particles and some other secondaries and it was argued that the characteristics of these leading fireballs are similar to the excited objects produced in deep inelastic lepton-hadron scattering.This assumption allowed us to calculate the average charged multiplicity in $p\bar{p}$ collision from ISR to SPS energies and we were moderately successful in reproducing the experimental data. As the momentum distribution of the leading particles was a bigger challenge compared to that for the other secondaries,we addressed this question.The leading particle (p or $\bar{p}$ in $p\bar{p}$ collision) may arise from single diffraction type events (rarely, double diffraction events) or from non-diffractive collisions. Assuming that in single-diffractive-events, a fireball is created with properties similar to the leading fireballs of non-diffractive events and using appropriate weight factors for single-diffractive and non-diffractive events, it was possible to calculate the proton (or antiproton) momentum distribution (Fig.2,3).

2. An interesting thing that was observed at the SPS was the flattening of the $< p_T >$ versus dN/dy curve at relatively high values ofdN/dy. (The variable y, called the rapidity, is related to the energy E and the longitudinal momentum p_L of a particle by $y = 1/2[ln(E + p_L)/(E - p_L)]$). This was interpreted by van Hove as the signal of QGP (Quark-Gluon-Plasma) formation, or more precisely, of the existence of a first-order phase transition between QGP and the hadron gas phase. However, we pointed out that a flattening of $< p_T >$ as a function of dN/dy is a natural consequence of the variation of k ('inelasicity') and of the KNO scaling

function in connection with the multiplicity distribution [7]. Saul Barshay gave a somewhat similar interpretation at about the same time (Fig.4).

3. In the mid eighties the idea was gaining ground that there is a strong first order phase transition between the QGP and the hadron gas. This was bolstered by some successes of the MIT bag model and of the associated equation of state. Accepting that the physical vacuum exerts a pressure on the plasma , we calculated the dimensions of the cylinder-shaped fireball, produced in e^+e^- annihilation, that would fit the experimental data on charged multiplicity etc. [8]. We found that the transverse dimension of the cylinder agrees, more or less, with that of the chromo-electric flux tube between a quark and an antiquark.(The flux tube was introduced by Andersson et al as a model of confinement [9]). Moreover we could, at the same time, get rid of a puzzling piece of the general solution, known as the simple wave, which appears in Landau's solution.

4. In 1983 Baym et al published a paper on ultra-relativistic heavy ion collisions [10]. This was an elaboration of a paper by Bjorken in which he assumed the Chiu-Sudarshan type solution of the equations of hydrodynamic motion [3]. On the basis of this paper we calculated the mass spectrum of dileptons ($\mu^+\mu^-$) produced in relativistic heavy ion collisions [11] (Figs. 5 and 6). In that work we assumed a second order phase transition between QGP and the hadron gas which was considered too mild at that time but recent lattice QCD data do not rule out this possibility. We also emphasized the role of pre-equilibrium emissions and made some crude estimates of it. A number of papers have been written on non-equilibrium emissions during the last ten years [12].

As the crude estimate of pre-equilibrium emission was rather unsatisfactory, we started looking for a statistical mechanical description of the time evolution of the system moving towards thermal equilibrium. The relaxation approximation was inadequate because the QGP starts far from equilibrium. However QCD implies that gluons should equilibrate among themselves rather quickly. This suggested the possibility of eliminating gluons via the concept of a heat bath. The momentum distribution of the quarks then evolve until equilibrium with the heat bath is attained. In fact the time evolution of the quark momentum distribution is given by a Fokker-Planck equation [13].

5. Recently we have focussed our attention on finite-size effects; in particular, the fluctuations brought about by the finiteness of the ratio of

the surface area to the volume of the fireball, or by the finite spatial extension of the hadrons [14]. Referring to the latter aspect, we have pointed out that there must occur a natural discretization of the solution of the hydrodynamic equations; this results in a stochastic non-linear mapping relation between the rapidities of adjacent hadrons. The implications of this mapping relation have not been explored fully. However stochasticity, both due to size variations and inelasticity (k) variations, give rise to values of factorial multiplicity moments compatible with experimental data.

5. Conclusion

The hydrodynamic model, both in its one dimensional and three dimensional versions, is a very well-studied approach towards the solution of the complex problem of high energy collisions. Although no longer favoured as a tool for the analysis of nucleon-nucleon collisions, it is still considered as one of the best and physically most transparent models for the analysis of heavy ion collisions. We can therefore anticipate a number of applications of this model in the coming years.

I would once again like to acknowledge that a considerable part of what I have done in this field stemmed from ideas and comments from Professor Banerjee. I take this opportunity to express my gratitude to him. I wish him many years of active life.

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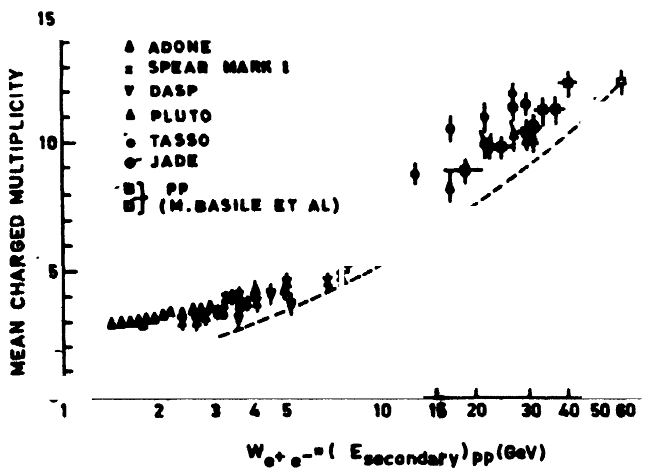

Fig.1. The energy dependence of the average charged multiplicity for pp collision is compared with data for e^+e^- annihilation. the points □ correspond to 'corrected' pp data (see text) plotted against the available energy E_{sec}. The dashed line shows the pp charged multiplicity as a function of the total cm energy W.

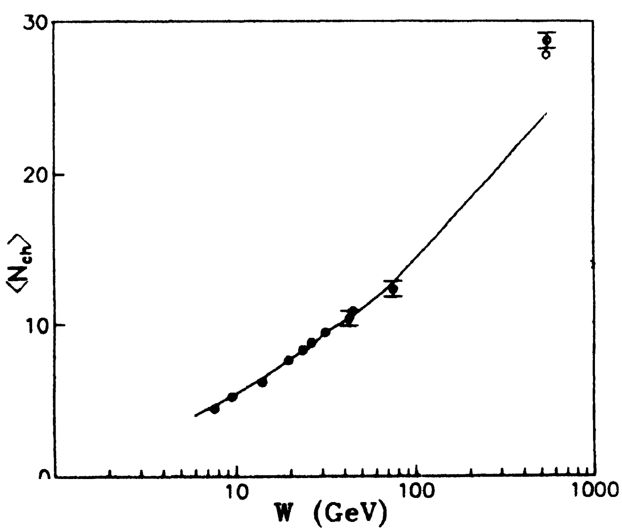

Fig.2. Plot of the average multiplicity of the charged hadrons vs. the total cm energy W in pp collisions. Closed circles represent the experimental multiplicities for the class of all (both diffractive and nondiffractive) inelastic collisions; the curve represents the corresponding multiplicities calculated on the basis of our model. Open circles with error bar represents the experimental nondiffractive multiplicity datum at the SPS collider energy. The open circle without error bar gives the corresponding theoretical multiplicity.

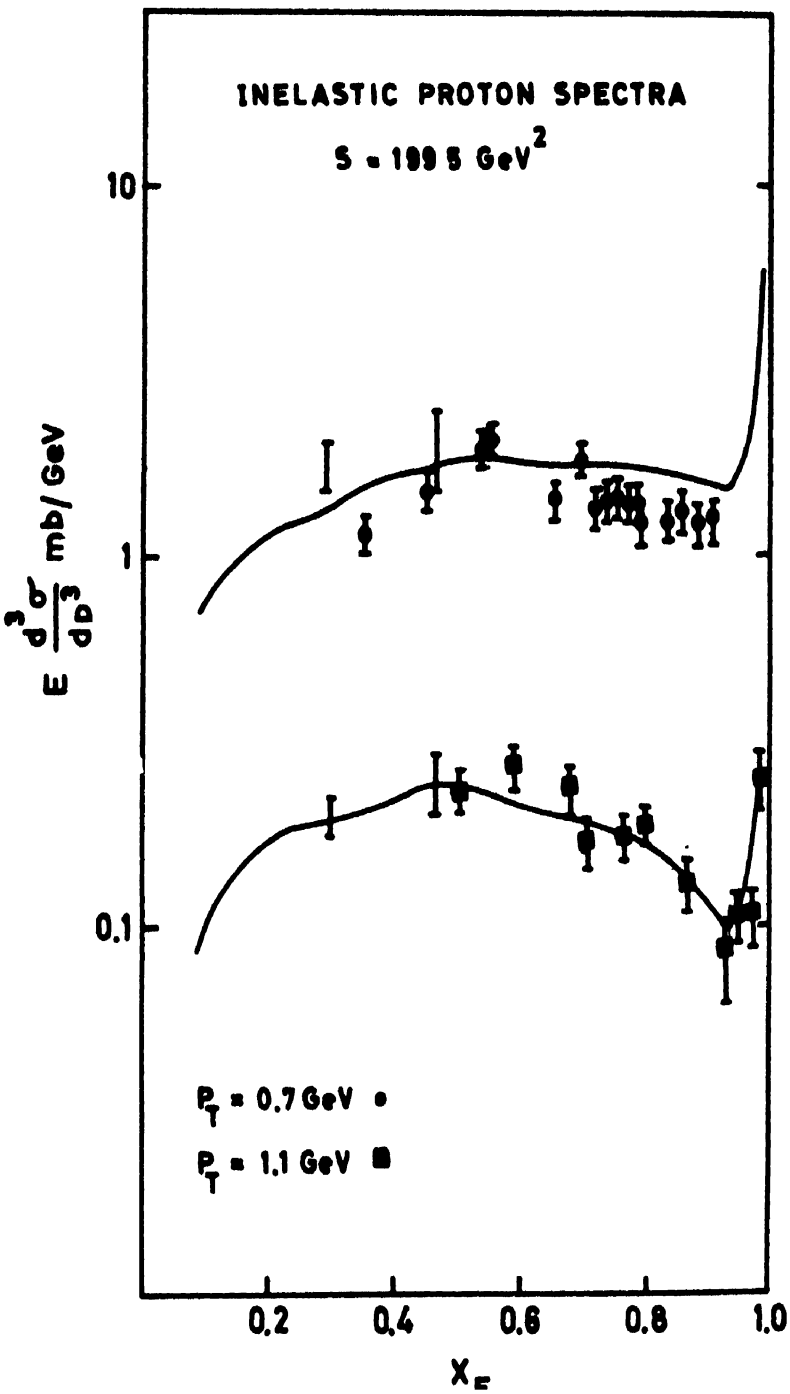

Fig.3. Inclusive cross-section for protons in pp collision at W 44.7GeV. Solid lines correspond to theoretical predictions.

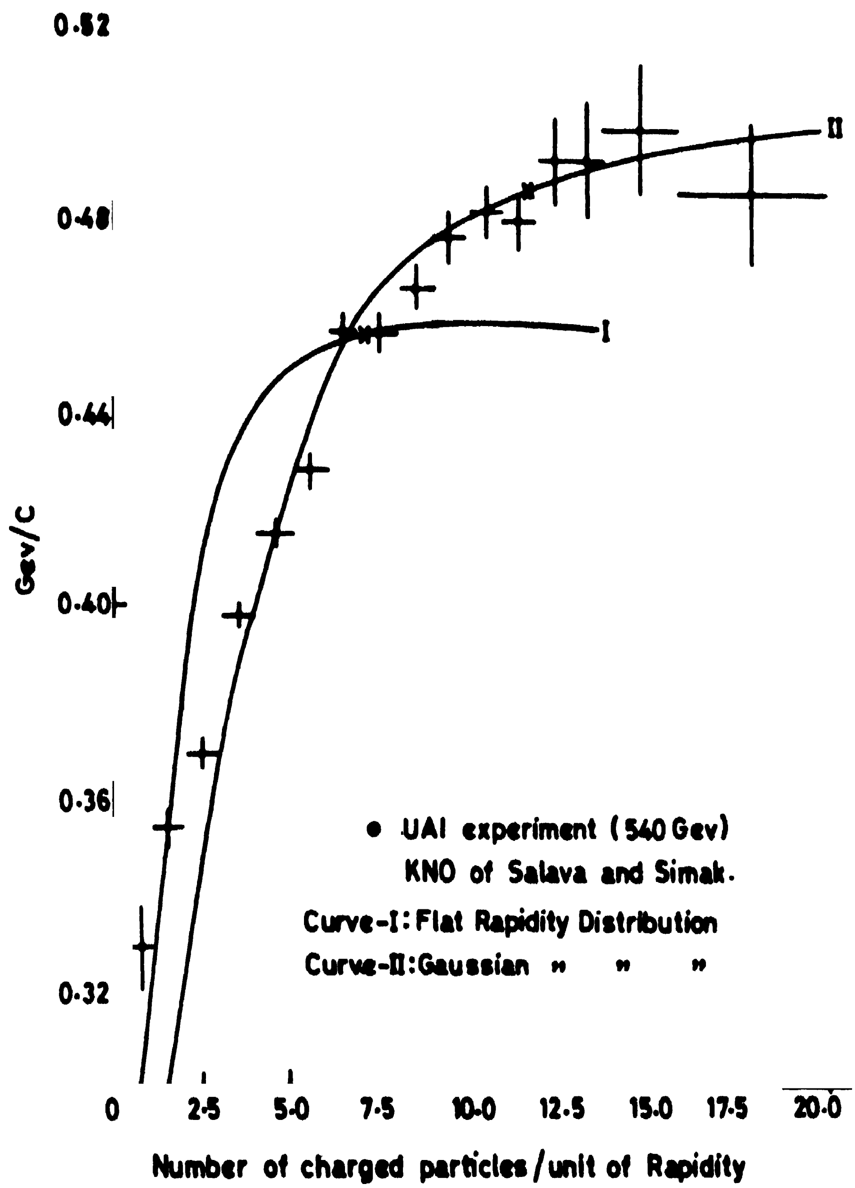

Fig.4. The mean transverse momentum of the charged hadrons at $W = 540 GeV$ as a function of charged multiplicity in the rapidity interval $|y| < 2.5$. The theoretical curves are drawn on the basis of the KNO scaling function of Salava and Simak. The overall normalisation parameter is fixed by reference to the point marked by a cross.

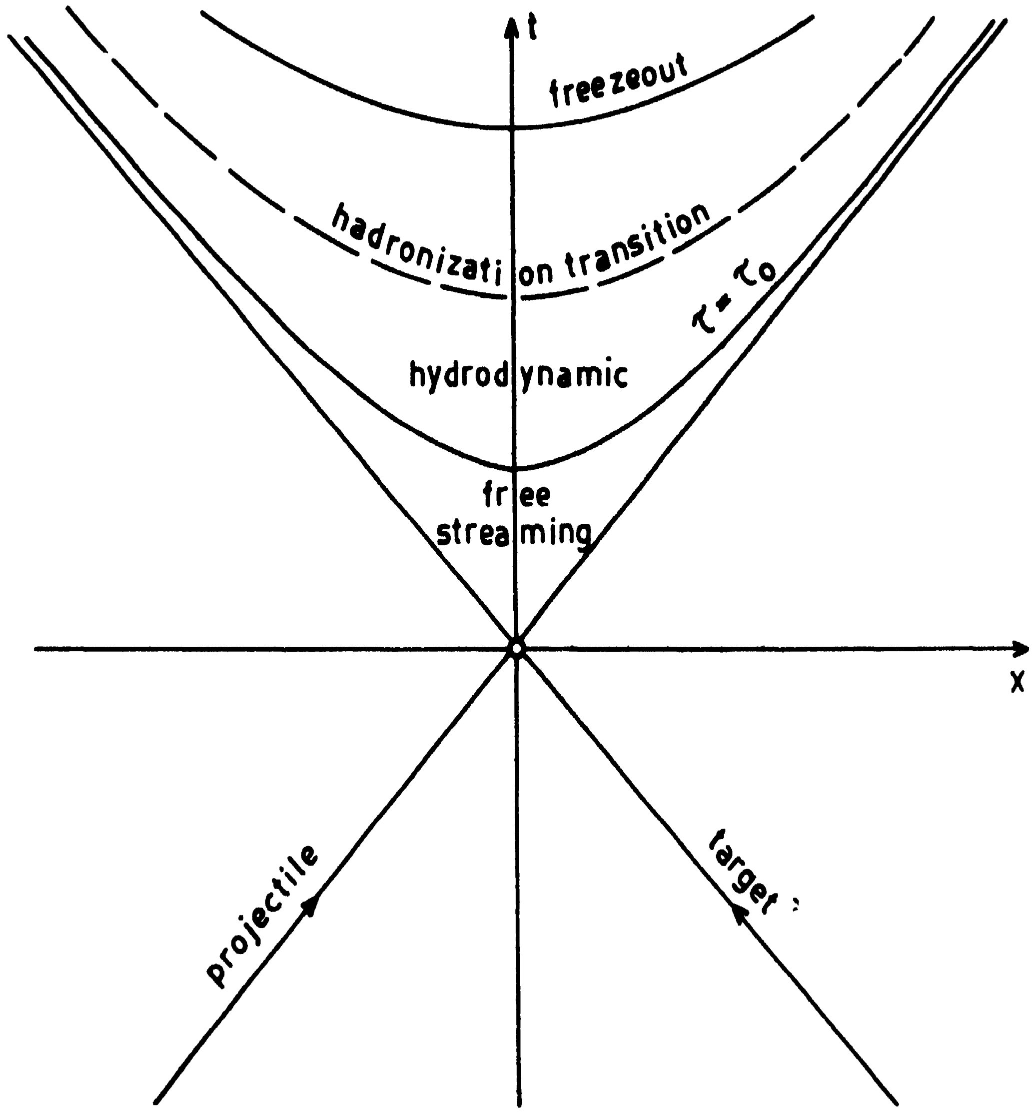

Fig.5. Space-time picture of a head-on collision between two heavy ions in the centre of mass frame, with the longitudinal thickness of the nuclei neglected. The lowest hyperbola represents the transition from free streaming to hydrodynamic behaviour, while the uppermost hyperbola represents the freezeout transition to free streaming hadrons. Also indicated as a dashed hyperbola is the hadronisation transition that will occur if a quark-gluon plasma is formed in the early stages of the collision.

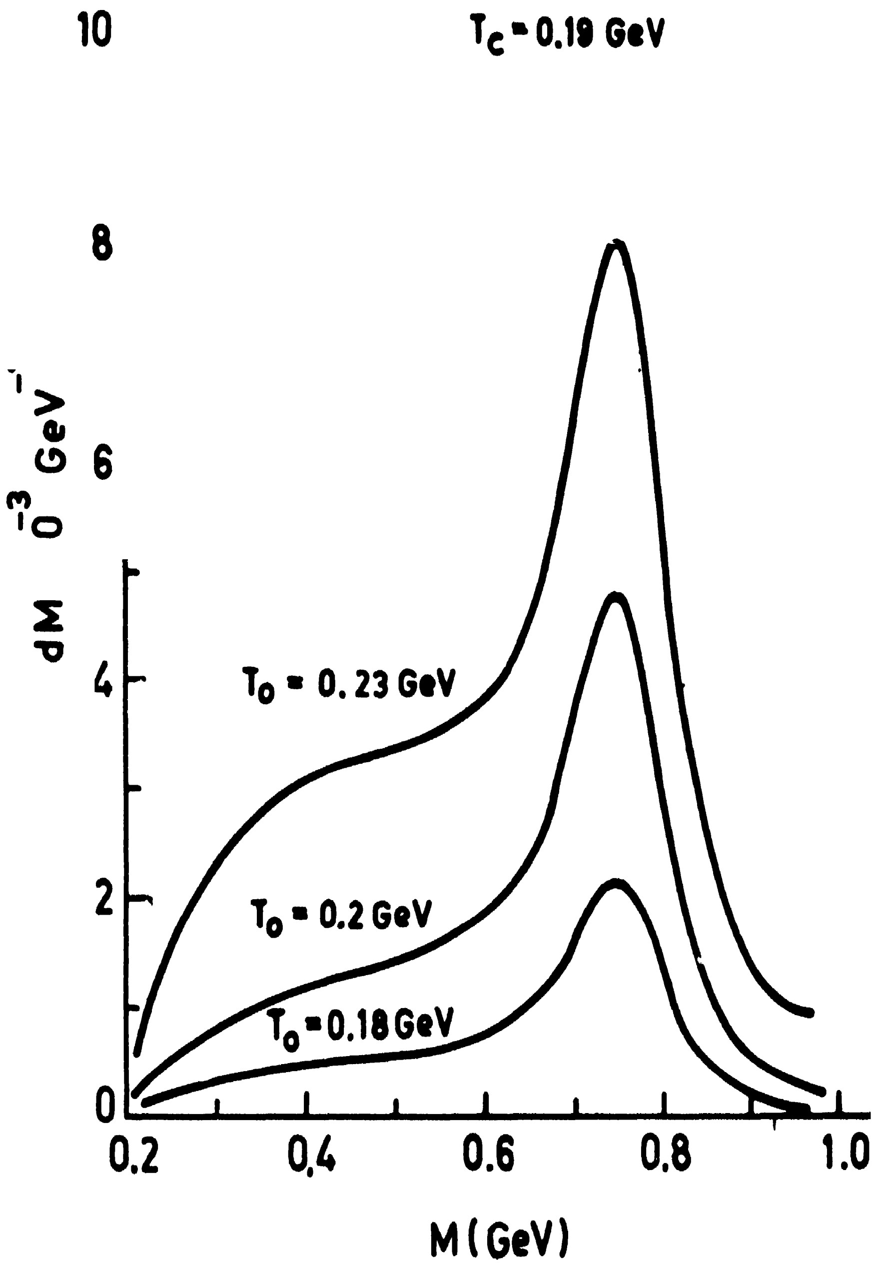

Fig.6. The dimuon mass spectrum for three initial temperatures T_0 =0.18, 0.20 and 0.23 GeV . In these calculations τ_0= 1 fm and T_c =0.19 GeV . The ions considered are those of Uranium.